Analysis of Power System Transient Stability of the Nigerian 330-kV Electricity Grid

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ORIGINAL RESEARCH

Abstract- Faults on power system are inevitable and can occur at any time resulting into transient instability. Transient instability can cause loss of synchronism and possible damage to power equipment and consumer loads. Therefore, this study analyzed the transient stability (TS) of the Nigerian 330-kV, 34-bus power network. Power flow and swing equations describing the system steady and transient states were respectively analyzed using Newton-Raphson and Runge-Kutta, fourth order numerical techniques to provide linear solution due to their non-linearity. Simulations were done and swing curves for different fault conditions obtained. The critical clearing time at which the simulated faults were cleared was determined. The results of load flow analyses revealed that buses 6, 10, 13, 14, and 17 with respective voltage magnitude of 0.937, 0.921, 0.938, 0.829 and 0.786 per unit violated voltage tolerance limit of 0.95 to 1.05 p.u. The system total active and reactive power losses were 57.65 MW and -70.62 MVar respectively. The obtained swing curve showed that for fault on bus 11 [line 9-11] generators 2, 3 and 7 lost synchronism after 1.0 s but other generators retained their stability. Similarly, for fault on bus 13 [line 10-14] generators 3 and 7 lost synchronism but other generators retained stability after 0.4 s. These results indicate that faults occurrence on generating stations of power system is unavoidable, but the clearance time must be short (few seconds) to avert total system collapse and loss of synchronism.

Keywords- Critical Clearing Time, Newton-Raphson method, Runge-Kutta, Swing Equation, Transient Stability.

1 INTRODUCTION

Globally, power systems are complex interconnected networks of components that involve generation, transmission and distribution of electricity and disturbances usually occur in these core segments of the systems. At the generation station, the synchronous generators are operated in synchronism with bus of the same voltage, phase sequence and frequency to generate electrical power (Latt, 2019). The ability of such generators to maintain synchronism under normal and abnormal operating conditions is called stability. However, if the system falls out of synchronism due to a disturbance whether small or large, its stability is affected and the degree of instability depends on the nature or severity of the disturbances and the initial operating conditions (Micheal et al. 2017).

Power instability has pronounced negative impact on effective system operation which may cause damage to electrical machines and consumer loads and if such fault is not cleared within shortest time, the system can deviate from steady-state and results into transient instability conditions. In order to maintain continuity of any power system and maximum power transfer to loads, it is necessary to ensure stable and synchronous operation under all system conditions (Mohammed et al. 2017; Hussain, 2012). Therefore, the need for transient stability analysis (TSA) becomes evident.

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power system transient stability studies.

Abbas et al. (2021) assessed a multi-machine power system’s transient stability using a load angle method in electrical transient analyzer program (ETAP) environment. The results revealed that faulted part should be rapidly isolated from the system to increase stability margin and hence, decrease possibility of damage. In a similar development, Micheal et al. (2020) examined the TS of a multi-machine power system in ETAP environment. The work revealed that those generators that were closer to the fault position experienced pronounced power deviation during contingency analysis. Mishra et al. (2020) examined power system transient stability by analyzing and comparing solutions of time domain, direct, and artificial intelligent methods. The work showed that delay was more pronounced in time domain than energy based direct method. Ochogwu et al. (2019) assessed the TS of the Nigeria 330-kV electric power transmission grid. The work showed that shunt capacitor application was able to improve system’s performance and faster recovery was observed when three phase fault was introduced. Ogboh et al. (2018) worked on transient stability analysis of the Nigerian power network for maintenance of steady flow. The resulting rotor angle-time relationships were examined for series of severe disturbances and result revealed that different swing curves were generated due to fault location and clearing time.

Mohammed et al. (2017) presented a novel approach for the analysis of a two-machines power system’s transient stability under fault condition using time domain method. The results showed that the system’s stability was sustained following the clearance of the fault which was found to be a function of rotor oscillation frequency. Raj and Jain (2016) compared different methods of transient stability assessment on single and multi-machine. The study revealed that the determination of critical clearing time of a single machine was straightforward while that of multi-machine was obtained through repeated time-domain simulations.

Judging from the above reviewed works, different studies had addressed different transient instability problems. With respect to the Nigeria scenario, both time-domain and direct methods had been applied for transient stability assessment. The two methods have complementary strengths and drawbacks. Time domain method can generate time response of all quantities and possesses unlimited modeling capability. It is also useful for detailed investigation of small and large disturbances in the system. But, the inherent quality features of a direct method including enhanced transient energy margin, fast response with limited modeling capability and suitability for first swing stability assessment render it the choice for this study. Unlike the approaches adopted in most of the reviewed works, this study employed direct coding for transient stability simulation on a practical large-scale Nigerian 330-kV, 34-bus power network.

### 3 Methodology

#### 3.1 Load Flow Formulation

Load flow provides information such as bus voltage magnitude and phase as well as the real and reactive power flow for analysis of power system steady-state conditions. For the load flow analysis in this study, a typical model of a power network of Fig. 1 was considered.

![Fig. 1: The typical \( j^{th} \) bus model of a power system (Adejumobi, et al., 2015).](image)

The complex power injected by the generating source into \( j^{th} \) bus is expressed as:

\[
S_j = P_j + jQ_j = V_j^* I
\]

Simplifying (1) results into (2) and (3) given for \( j = 1,2,3,....n \) as:

\[
P_j = \sum_{k=1}^{n} |V_k| |V_j| \cos(\theta_{jk} - \delta_k - \delta_j)
\]

\[
Q_j = \sum_{k=1}^{n} |V_k| |V_j| \sin(\theta_{jk} - \delta_k - \delta_j)
\]

Where; \( P_j, Q_j, |V_j| \) and \( \delta_j \) are the real power, reactive power, voltage magnitude and phase angle referring to bus \( j \) respectively, while \( |V_k| \) and \( \delta_k \) are the voltage magnitude and phase angle respectively referring to bus \( k \). Equations (2) and (3) are called the static load flow equation. As a result of the non-linearity of (2) and (3), Newton-Raphson numerical method was used to linearize the two equations because of its accuracy and faster convergence ability. The linearized Newton-Raphson load flow equation in polar form is expressed as:

\[
\begin{bmatrix}
\Delta P_1^{(m)} \\
\vdots \\
\Delta P_n^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial P_n}{\partial V_1} & \cdots & \frac{\partial P_n}{\partial V_n}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_n
\end{bmatrix}

\]

\[
\begin{bmatrix}
\Delta Q_1^{(m)} \\
\vdots \\
\Delta Q_n^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial Q_n}{\partial V_1} & \cdots & \frac{\partial Q_n}{\partial V_n}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_n
\end{bmatrix}

\]

\[
\begin{bmatrix}
\Delta S_1^{(m)} \\
\vdots \\
\Delta S_n^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
\Delta P_1^{(m)} & \Delta Q_1^{(m)} \\
\vdots & \vdots \\
\Delta P_n^{(m)} & \Delta Q_n^{(m)}
\end{bmatrix}
\begin{bmatrix}
\Delta P_1^{(m)} \\
\vdots \\
\Delta P_n^{(m)}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta P_1^{(m)} \\
\vdots \\
\Delta P_n^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial P_n}{\partial V_1} & \cdots & \frac{\partial P_n}{\partial V_n}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_n
\end{bmatrix}

\]

\[
\begin{bmatrix}
\Delta Q_1^{(m)} \\
\vdots \\
\Delta Q_n^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial Q_n}{\partial V_1} & \cdots & \frac{\partial Q_n}{\partial V_n}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_n
\end{bmatrix}

\]

\[
\begin{bmatrix}
\Delta S_1^{(m)} \\
\vdots \\
\Delta S_n^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
\Delta P_1^{(m)} & \Delta Q_1^{(m)} \\
\vdots & \vdots \\
\Delta P_n^{(m)} & \Delta Q_n^{(m)}
\end{bmatrix}
\begin{bmatrix}
\Delta P_1^{(m)} \\
\vdots \\
\Delta P_n^{(m)}
\end{bmatrix}

\]
The residuals power \( \Delta P_j^{(z)} \) and \( \Delta Q_j^{(z)} \) represent the difference between the specified and determined values at each iteration which are given by

\[
\Delta P_j^{(z)} = P_j^{(z)} - P_j^{(z)}
\]

\[
\Delta Q_j^{(z)} = Q_j^{(z)} - Q_j^{(z)}
\]

Where;

\( z \) is the iteration count,

\( \Delta P_j^{(z)} \) and \( \Delta Q_j^{(z)} \) are the changes in the real and reactive power at the iteration \( z \), respectively.

\( P_j^{(z)} \) and \( Q_j^{(z)} \) are the specified real and reactive power supplies at bus \( j \), respectively.

\( P_j^{(z)} \) and \( Q_j^{(z)} \) are the calculated real and reactive power demands at bus \( j \), respectively.

The new values for the bus voltages and corresponding phase angles are obtained using

\[
\delta_j^{(z+1)} = \delta_j^{(z)} + \Delta \delta_j^{(z)}
\]

\[
|V_j^{(z+1)}| = |V_j^{(z)}| + \Delta |V_j^{(z)}|
\]

where \( \delta_j^{(z)} \) is the new bus voltage angle, and \( |V_j^{(z+1)}| \) is the new bus voltage magnitude. This process is continued until residuals \( \Delta P_j^{(z)} \) and \( \Delta Q_j^{(z)} \) satisfies the tolerance limits specified as

\[
|\Delta P_j^{(z)}| \leq \epsilon
\]

\[
|\Delta Q_j^{(z)}| \leq \epsilon
\]

where \( \epsilon \) is tolerance or accuracy level chosen for load busses.

The power loss in line \( j-k \), which is the algebraic sum of the power flow is obtained as

\[
S_{lk} = S_{j\beta} + S_{ij}
\]

where \( S_{LK} \) is the generated apparent power injected into the \( j^{th} \) bus, \( S_{j\beta} \) is the apparent load power flowing out of the \( j^{th} \) bus, and \( S_{ij} \) is the apparent transmitted power flowing out of the \( j^{th} \) bus.

### 3.2 Swing Equation Formulation for a Multi-Machine Power Network

Under steady state conditions without any disturbance, the dynamics of the rotor are expressed by (12) to (15) (Ogboh, 2018).

\[
T_m = T_m
\]

\[
T_e = T_m - T_e
\]

\[
\frac{H}{\pi f_e} \frac{d^2 \delta_m}{dt^2} = P_m - P_e
\]

\[
\frac{H}{180 f_e} \frac{d^2 \delta_e}{dt^2} = P_m - P_e
\]

Where:

\( T_m \) and \( T_e \) are the mechanical torque, electrical torque output, and the net accelerating torque all in Newton-meters respectively, \( H \) is the machine’s inertia constant, \( \delta_m \) is the system’s phase angle, \( f_e \) is the system’s output’s fundamental frequency in Hertz.

Equation (15) is a nonlinear second order differential equation and it is called swing equation which is expressed as two first order differential equations by Shereefdeen et al. (2016) as:

\[
\frac{d\delta}{dt} = (\omega_0 - \omega) = \Delta \omega
\]

\[
\frac{d\omega}{dt} = \frac{\pi f_e}{H} (P_m - P_e)
\]

### 3.2.1 Solution of Swing Equation using Runge-Kutta, Fourth Order Technique

The analysis of the transient stability requires the solution of swing equations. For this work, Runge-Kutta, fourth-order numerical method is used for solution of (15) as expressed by (16) and (17).

Applying Runge-Kutta, 4th order routine to (16) and (17) result into,

\[
\delta = \delta_0 + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)
\]

\[
\omega = \omega_0 + \frac{1}{6} (I_1 + 2I_2 + 2I_3 + I_4)
\]

where \( \delta \) is the rotor angle and \( \omega \) is the angular velocity, \( K_1, K_2, K_3, K_4 \) and \( I_1, I_2, I_3, I_4 \) are independent variable; \( \delta_0 \) and \( \omega_0 \) are change in \( \delta \) and \( \omega \) respectively. Using \( \delta \) and \( \omega \) as initial values for the succeeding time step (Padha and Mishra, 2015; Sharma, 2014). Equations (18) and (19) represent the numerical solutions of (16) and (17) that are used for simulation of the test network to estimate the critical clearing angle, which denoted as \( \alpha \) and corresponding critical clearing time \( (t_{cr}) \).

### 3.3 Test System

The test network for this study is the Nigerian 330-kV, 34-bus electricity network. The one-line diagram of the network is shown in Figure 2 while bus, generator, and branch parameters were presented in the appendices (A) to (C).
3.4 Simulation Software

Computer programs are very useful tools to perform different analysis on multi-machines of complex power systems which makes it effective for the researchers to have a better assessment of complex power system. Therefore, MATLAB R2018a software was used for the development of the code used in this study.

4 RESULTS AND DISCUSSION

4.1 Load Flow Analysis Results

The voltage profile of the Nigerian 34-bus system for pre-fault load flow analysis is shown in Fig. 3. The results revealed that buses 6, 10, 13, 14, and 17 with respective voltage magnitude of 0.937, 0.921, 0.938, 0.829 and 0.780 per unit violated voltage tolerance limit of 0.95 to 1.05 per unit. The systems total active and reactive power losses were 57.62 MW and -70.62 MVar respectively.

4.2 Results of Transient Stability Analysis

The simulation results when balanced three-phase short circuit fault was introduced on buses 11 and 13 of the Nigeria 330-kV, 34-bus electricity grid are presented in Figures 4 and 5, respectively. such as faulted bus, the faulty line number to be removed and fault clearing time are specified.

The results obtained from the TSA carried out on Nigeria 330-kV power system show that for fault on bus 11, the system is stable with clearing time of 0.05 second in Fig. 4(a). However, the rotor angles difference of generators 2, 3 and 7 loses synchronism at the clearing value of 1.0 second, while generators 4, 5, 6, 8, 9, 10 and 14 maintain their synchronism. This can be inferred from Fig. 4(b).

Figure 5(a) shows the result obtained when fault occurs on bus 13, the system is stable with fault clearing time of 0.2 second; as the clearing time increases to 0.4 second, the rotor angles difference of two generators: 3 and 7 loss synchronism, while generators 2, 4, 5, 6, 8, 9, 10 and 14 remain stable. These observations are obvious in Figure 5(b).

The results obtained from Odu et al. (2023) Micheal et al. (2020), Mohammed et al. (2017), and Shereefdeen et al. (2016) among others who have worked on TS of power systems validate the results of this study, where it is submitted that delay in faults clearing, after critical value, on a power system network can lead to system instability and system collapse.
5 Conclusion

The analysis of transient stability of power system was carried out in this study where it is found that critical clearing time for fault on the system is very important for system stability. The shorter the clearance times, the better the system in terms of stability. It is concluded that fault critical clearing time is a determining factor to maintain power system within the stability limits. An increase in clearing times from the critical values result into loss of synchronism. This study recommends that post fault load flow of the test system be carried out for complete transient stability assessment of the considered network in the future. For power system transient stability improvement, power electronics technology such as flexible alternating current transmission system (FACTS) devices’ incorporation into transmission network is also recommended.

APPENDICES

Appendix A. Bus Parameters of the Nigeria 330-kV, 34-Bus Power System Network

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Bus Type</th>
<th>$P_d$ (MW)</th>
<th>$Q_d$ (Mvar)</th>
<th>$V_m$ (p.u)</th>
<th>$V_{m\max}$ (p.u)</th>
<th>$V_{m\min}$ (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>1.06</td>
<td>1.05</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>40.00</td>
<td>-10.00</td>
<td>1.0</td>
<td>1.05</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
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<td>1.05</td>
<td>0.95</td>
</tr>
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</tr>
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<td>1.05</td>
<td>0.95</td>
</tr>
<tr>
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<td>1.05</td>
<td>0.95</td>
</tr>
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<td>0.00</td>
<td>1.02</td>
<td>1.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Appendix B. Generator Parameters of the Nigeria 330-kV, 34-Bus Power System Network

<table>
<thead>
<tr>
<th>Bus No</th>
<th>$P_g$ (MW)</th>
<th>$Q_g$ (Mvar)</th>
<th>$Q_{r_{max}}$ (Mvar)</th>
<th>$Q_{r_{min}}$ (Mvar)</th>
<th>$V_r^*$ (p.u.)</th>
<th>$R_0$ (p.u)</th>
<th>$X_0$ (p.u)</th>
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Appendix C. Branch Parameters of the Nigeria 330-kV, 34-Bus Power System Network

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ACKNOWLEDGEMENT
The authors would like to sincerely appreciate the words of encouragement from the Acting Head, Department of Electrical and Electronics Engineering, Federal University of Agriculture, Abeokuta, Dr. Amusa, K. A., and Engr. Mathew, S. of the same Department and others for their invaluable suggestions and supports.

REFERENCES


