Derivation of the Rainfall Intensity, Duration and Frequency Equations for Makurdi, Nigeria

1Kehinde A. Bolorunduro, 2Opeyemi K. Olayanju, and 3Isaiah A. Oke
1Department of Civil Engineering, Federal University, Oye- Ilé-Ife, Nigeria
2Department of Civil Engineering, Redeemer University, Ededeji, Nigeria
3Department of Civil Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria

kolorunduro@gmail.com; olayanjuro@run.edu.ng; okeia@oauife.edu.ng

Received: 17-AUG-2022; Reviewed: 14-SEP-2022; Accepted: 18-DEC-2022
https://doi.org/10.46792/fuoyejet.v7i4.915

ORIGINAL RESEARCH

Abstract- Information and adequate data on intensity–duration–frequency of rainfall are regularly required for a variation of hydrologic, environmental and hydraulic applications. This paper presents rainfall intensity-duration-frequency equations for Makurdi, Nigeria. Rainfall intensities from Makurdi were used to establish empirically derived constants for about thirteen different equations. These equations were evaluated statistically using analysis of variance (ANOVA), total error, model of selection criterion (MSC), Coefficient of Determination (CD), Correlation coefficient (R) and Akaike Information Criterion (AIC), with the main objective of selecting the best equation for the location. The study revealed that rainfall intensity can be expressed in terms of either the amount of rainfall time only, frequency or both duration of the rain and time of return with empirically derived constants. Statistical evaluation revealed that the correlation coefficients of equations for Makurdi were between 0.896 and 0.894, respectively. Model 12 was discovered to be most accurate model for prediction of rainfall amount as against model 15 presented in literature. The model had the highest correlation coefficient for Makurdi. According to findings, functions of the duration of rainfall, return time, and empirically derived constants are the best functions, which explains the severity of the rain the best based on the values of MSC, CD, AIC and R.

Keywords- Analysis of variance, Coefficient of Determination, Frequency analysis, Model, Rainfall intensity,

1 INTRODUCTION

Rainfall frequencies of different intensities and durations are used extensively in the design and management of many environmental and water resources projects, which comprise natural hazards, due to extreme rainfall events. The most mutual frequency analysis entails developing a relationship between rainfall intensity duration and return period (Adamowski and Bougadis, 2003). Increasing cases of enormous and destructive floods, together with vulgar floods and landslides are the impacts of global warming (Shukor et al., 2020). Estimating the quantity of runoff-rainfall intensity is required in the rational equation. A lot of researchers have worked on the development and evaluation of rainfall intensity equations.

Aho et al. (2019) presented and analysed Makurdi rainfall data from 1955 to 2015. The study revealed that extreme rainfall events tally with flood occurrence. It was stated that the total annual rainfall is not necessarily the most important cause of increased flood frequency. Agada et al. (2016) evaluated the characteristics of rainfall in Makurdi through rainfall data between 1985 and 1987.

It was concluded that the generally high erosivity values are pointers to the compelling need for soil protective covers in particular and the integration of other serious conservation measures as key strategies for flood control and sustainable production in this agro-ecological zone. Figure 1 presents a typical relationship between probability and rainfall intensity. Some of these developments are established in literature such as Bell (1969), Chen (1983), Koutsosyianni et al. (1998); Maria-Carmen (2001); Nhat et al. (2006), Raiford et al. (2007), Endrenyi and Imbeah (2009), Perez-Latorre et al. (2010); El-Sayed (2011), Awadallah et al. (2011; 2017); Karahan (2012); Wagesho and Claire (2016); Li et al. (2017); Melillo et al., (2018); Al-Zahrani (2018); Şen (2019).

Over the past three decades, researchers worldwide have analysed the Spatio-temporal development of rainfall and its behaviour due to the impending climate change effects. It has been reported that there are indications of increasing rainfall in certain parts of the world. On the other hand, there are decreasing patterns of rainfall in other parts of the world (Assouline et al., 2007; AlHassoun, 2011; Elsebaie, 2012; Dottori et al., 2016). Therefore, local scale studies of rainfall intensity and the presentation of accurate equations for the prediction of rainfall intensity are necessary as this information is required as the main input for modelling hydrological behaviours, determination of storm discharge and flow rate for sewerage systems. The purpose of this study is to evaluate the performance accuracy of rainfall duration frequency equations in relation to the practical values in line with the criteria for hydrological analysis and utilization of water resources and environmental projects.

© 2022 The Author(s). Published by Faculty of Engineering, Federal University Oye-Ekiti. This is an open access article under the CC BY NC license. (https://creativecommons.org/licenses/by-nc/4.0/)

http://doi.org/10.46792/fuoyejet.v7i4.915

Section E: CIVIL ENGINEERING & RELATED SCIENCES

Can be cited as:


© 2022 The Author(s). Published by Faculty of Engineering, Federal University Oye-Ekiti. This is an open access article under the CC BY NC license. (https://creativecommons.org/licenses/by-nc/4.0/)

http://doi.org/10.46792/fuoyejet.v7i4.915

http://journal.engineering.fuoye.edu.ng/
2 MATERIALS AND METHOD

The annual maximum rainfall duration data series were obtained by selecting the maximum amount of rainfall for each year and for 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 1440 durations (minutes) for the 30 years period (1979 to 2009) from literature as presented by Isikwue et al. (2012). Figure 2 presents the rainfall intensity and duration information used. A total of 210 rainfall intensity data were established from the source and the data was used to evaluate selected standard rainfall duration frequency equations available in the literature (13 equations). The rainfall intensities were ranked, return period and probability of occurrence were determined using Weibull expression as shown in equations 1 and 2 (Mays, 2011; Maidment, 1993; Chin, 2000):

\[ T_m(x) = \frac{n+1}{m} \]  
(1)

\[ p_m(x) = \frac{1}{T_m} \]  
(2)

The constants in these selected equations were fixed using Microsoft Excel Solver. These equations were used to calculate rainfall intensity through the use of Microsoft Excel Solver and these rainfall intensities were compared with observed rainfall intensity. Calculated values of rainfall intensity were evaluated statistically using analysis of variance (ANOVA), total error, Model of selection criterion (MSC), Coefficient of Determination (CD), Correlation coefficient (R) and Akaike Information Criterion (AIC). The AIC was derived from the Information Criterion of Akaike (Akaike, 1974). It allows a direct comparison among models with a different number of parameters. The AIC presents the information on a given set of parameter estimates by relating the coefficient of determination to the number of parameters. MSC indicates higher accuracy, validity and a good fit of the methods. MSC was computed using equation (3) as follows:

\[ MSC = \ln \left( \frac{\sum_{i=1}^{n}(Y_{obs}-Y_{cali})^2}{\sum_{i=1}^{n}(Y_{obs}-\bar{Y})^2} \right) - \frac{2p}{n} \]  
(3)

Where, \( Y_{obs} \) is the rainfall intensity from the experimental study; \( Y_{cali} \) is the average rainfall intensity from the experimental study; \( p \) is the total number of fixed parameters to be estimated in the methods; \( n \) is the total number of rainfall intensity calculated, and \( Y_{cali} \) is the rainfall intensity calculated using the methods.

The AIC was determined using Equation (4) as follows:

\[ AIC = n \left( \ln \sum_{i=1}^{n}(Y_{obs}-Y_{cali})^2 \right) + 2p \]  
(4)

The coefficient of determination (CD) can be interpreted as the proportion of expected data variation that can be explained by the obtained data. Higher values of CD indicate higher accuracy, validity and good fitness of the device. CD and correlation coefficient can be expressed as given in equations (5) and (6):

\[ CD = \frac{\sum_{i=1}^{n}(Y_{obs}-Y_{cali})^2 - \sum_{i=1}^{n}(Y_{obs}-\bar{Y})^2}{\sum_{i=1}^{n}(Y_{obs}-\bar{Y})^2} \]  
(5)

\[ R = \sqrt{\frac{\sum_{i=1}^{n}(Y_{obs}-Y_{cali})^2}{\sum_{i=1}^{n}(Y_{obs}-\bar{Y})^2}} \]  
(6)

Relationships between rainfall intensity, return period and probability of these rainfall intensities determined by using the Weibull technique were presented. Isopluvial maps of the best two equations were determined to aid hydrologists, environmental engineers, developers and planners in the design as well as management of stormwater.
Figure 3 presents the summary of Microsoft Excel Solver (MES) procedures. MES was used for the determination of these empirically derived parameters based on availability at no additional cost. The procedure used for Microsoft Excel solver can be summarized as follows:

i. Excel solver was added in Microsoft Excel,
ii. Target of the numerical analysis \((K_p - K_t)^2 = 0\), operation and changing cells were set,
iii. Where; \(K_p\) is the observed rainfall intensity and \(K_t\) is the calculated rainfall intensity; and
iv. Microsoft Excel Solver was allowed to iterate at 200 iterations with 0.005 tolerance (Figure 3).

The procedures (Figure 3) revealed that there are key steps to be conducted as follows:

a. Set the target
b. Select the operation (zero out of the three operations available)
c. Highlight the changing cells and variables
d. There are no limitations or constraints.

3 RESULTS AND DISCUSSION

Figure 2 presents rainfall-duration intensity frequency data as obtained from the source. From the Figure, the highest rainfall- duration-intensity frequency occurred when the duration time was 5 minutes in the year 1979 (year 1) and the lowest rainfall- duration intensity frequency occurred when the duration was 1440 minutes in the 30th year (2009). Figure 4 presents the relationship between probability, return period and rainfall intensity computed. Figure 5 shows the relationship between duration, rainfall intensity and return period using Isikwue et al., 2012 technique. These results revealed that the duration of heavy rainfall was shorter than duration of light or medium rainfalls. These observations are in agreement with previous studies and researches (Chow et al., 1988; Vyver and Demaree, 2010; Awadallah et al., 2011; Akin et al., 2011; Tung and Wong, 2014; Rahman, 2015; Zope et al., 2016; Al-Amri and Subyani, 2017; El-Tantawi et al., 2021; Susilowati and Alijahbana, 2022).

Table 1 shows the result of ANOVA of the rainfall-duration intensity frequency. From the Table, the \(F_{3, 170} = 7.59\) and \(p = 4.40 \times 10^{-20}\) for analysis of the rainfall-duration intensity frequency between the years. This result revealed that there was a significant difference between rainfall- duration-intensity frequency values within the years at 95 % confidence level \((p < 0.05)\). In the same Table 1, presents results of ANOVA of rainfall-duration-intensity frequency within the duration period of the rainfall. The Table shows that \(F_{3, 170} = 253.12\) and \(p = 8.70 \times 10^{-27}\) for analysis of the rainfall-duration-intensity frequency between the duration of the rainfall. This result revealed that there was a significant difference between rainfall-duration-intensity frequency values within these durations at 95 % confidence level \((p < 0.05)\). Table 2 provides information on the values of empirically derived parameters.

From the Table, there are four empirically derived parameters \((C, m, n\) and). The Table revealed that “\(C\)” ranges from 48.341 to 17627707.329, “\(m\)” was between 0.056 and 58067360.414, “\(n\)” was in the form of -0.213 to 116.547 and the value of “\(a\)” was between 0.548 and 39.831. These empirically derived parameters revealed that these rainfall intensity parameters are functions of the formulae (equations) and the other dependent variables. ASCE (1996) reported that there are two models that are common in America. The models are as follows:

a. \(I = \frac{c}{t_a + m} \) \hspace{1cm} (7)

b. \(I = C(t_a + m)^n \) \hspace{1cm} (8)
ASCE (1996) stated that the values of empirically derived parameters in equation (7) are C was in the range of 51.6 to 315.5, “n” has been in the range of 0.63 and 0.97 and “m” has been in the range of 2.06 and 13.90. In this study, C was found to be 86.352, “n” was 0.211 and “m” was 0.078. ASCE (1996) stated that the values of empirically derived parameters in equation (8) are C was in the range of 27.7 to 249.7, “n” has been in the range of 1.15 and 10.50 and “m” has been in the range of 0.51 and 0.84. In this study, C was found to be 86.234, “n” was -0.213 and “m” was 0.056. These values show that C, m and n are functions of locations and other factors.

Table 3 provides the results of ANOVA for the effects of the models on the empirically derived parameters. These observations and statistical establishment that there is a significant difference agree with literature such as Maidment (1993); ASCE (1996); Bell (1969); Chow et al. (1988); Mohamadreza et al., 2008Mays (2011); Vivekanandan 2013; Mirohesseni et al., 2013 Hingray et al. (2015); Massimo et al. (2018); Kwak et al. (2020); Fayez (2020); Kareem et al. (2022).

Table 3 presents statistical evaluation of each of the models. The Table revealed that the total error ranging from 191899.83 to 1276827.82, AIC was in the range of 2560.59 and 2958.58; and SC was in the form of 2570.63 to 2968.62. The highest error, AIC and SC originated from model 11, which indicated that the accuracy of the model was the lowest. The model states that rainfall intensity is directly proportional to the return period and partially constant and inversely proportional to the power of rainfall intensity computed using model 11 were very high in magnitude compared to the experimental values of rainfall intensity. In terms of R, these models can be classified into three main groups as follows:

a. The first class involves models with R-value greater than 0.88 (R>0.88). These models are models 1, 4, 5 and 12. These indicated that these four (4) models can predict the experimental rainfall intensity better than other models.
b. Second category involves models with the value of R greater than 0.80 but less than 0.88 (0.80 < R < 0.88). These models are models 9, 14 and 15. These indicated that these three (3) models can predict the experimental rainfall intensity above average.
c. Third category involves models with a value of R less than 0.80. This involves models 2, 3, 6, 7, 8, 10, 11, and 13. These indicated that these eight (8) models can predict the experimental rainfall intensity below average.
d. A further classification, revealed that Models 4, 5 and 12 had R of 0.894, 0.892 and 0.896 respectively.

These are presented as Models 4,5 and 12.

- a. Model 4
  \[ I = \frac{25919.7752^{0.446}}{(t_d + 1.16547)} \]

- b. Model 5
  \[ I = \frac{1337.147^{0.441}}{(t_d + 0.3055 + 1.812)} \]

- c. Model 12
  \[ I = \frac{281.111^{0.445}}{(t_d + 39.831)^{0.447}} \]

From these evaluations, three models performed better than the other models, which indicated that rainfall intensity is directly proportional to the power of return period, and inversely proportional to the power of duration of the rainfall and partially constant. Figures 6 and 7 present isovspual maps of models 12 and 4.

Figures 6 & 7 revealed that rainfall intensities of lower durations are with lower return period and rainfall intensities of higher durations are with higher return period. These observations are in agreement with literature (Maidment, 1993; ASCE, 1996; Bell, 1969; Chow et al.,1988; Mohamadreza et al., 2008; Mays, 2011; Vivekanandan, 2013; Mirohesseni et al., 2013 Hingray et al., 2015; Massimo et al., 2018; Kwak et al., 2020; Fayez, 2020; Kareem et al., 2022) and indicated that rainfall of higher magnitude are frequently occurred but must be considered in the design of storm drains.

### Table 1. The result of ANOVA of the Rainfall- Duration Intensity Frequency

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Square</th>
<th>Degree of freedom</th>
<th>Mean Sum of Square</th>
<th>F-Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the years</td>
<td>147235.3</td>
<td>34</td>
<td>4330.449</td>
<td>7.593873</td>
<td>4.4 x 10^{-20}</td>
</tr>
<tr>
<td>Within the duration of the</td>
<td>721710.3</td>
<td>5</td>
<td>144342.1</td>
<td>253.118</td>
<td>8.7 x 10^{-77}</td>
</tr>
<tr>
<td>rainfall</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>96943.46</td>
<td>170</td>
<td>570.2556</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>965889</td>
<td>209</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 3: Procedure for using Microsoft Excel Solver in the computation of the rainfall intensity

Fig. 4: Relationship between probability, return period and rainfall intensity computed

Fig. 5: Relationship between return period, duration and rainfall intensity as presented in Isikwue et al. (2012)
Table 2. Values of C, m, n and a for all the rainfall intensity determination models

<table>
<thead>
<tr>
<th>Model number</th>
<th>Model Equation</th>
<th>C</th>
<th>m</th>
<th>n</th>
<th>a</th>
<th>Actual Model Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>( I = \frac{C T_r^m}{t_d^n} )</td>
<td>48.341</td>
<td>0.412</td>
<td>0.027</td>
<td>( I = \frac{48341 \cdot t_r^{0.412}}{t_d^{0.027}} )</td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>( I = \frac{C}{t_d + m} )</td>
<td>86.234</td>
<td>0.056</td>
<td>0.213</td>
<td>( I = \frac{86234}{t_d + 0.056} )</td>
<td></td>
</tr>
<tr>
<td>Model 3</td>
<td>( I = \frac{C}{t_d + m} )</td>
<td>86.352</td>
<td>0.078</td>
<td>0.211</td>
<td>( I = \frac{86352}{t_d + 0.078} )</td>
<td></td>
</tr>
<tr>
<td>Model 4</td>
<td>( I = \frac{C T_r^m}{t_d + n} )</td>
<td>5919.775</td>
<td>0.446</td>
<td>116.547</td>
<td>( I = \frac{5919.775 \cdot t_r^{0.446}}{t_d + 116.547} )</td>
<td></td>
</tr>
<tr>
<td>Model 5</td>
<td>( I = \frac{C T_r^m}{t_d + a} )</td>
<td>133.714</td>
<td>0.461</td>
<td>1.305</td>
<td>1.812</td>
<td>( I = \frac{133714 \cdot t_r^{0.461}}{t_d + 1.305} )</td>
</tr>
<tr>
<td>Model 6</td>
<td>( I = \frac{C}{t_d} )</td>
<td>86.234</td>
<td>0.213</td>
<td>( I = \frac{86234}{t_d} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 7</td>
<td>( I = \frac{C}{t_d + m} )</td>
<td>86.234</td>
<td>0.056</td>
<td>-0.213</td>
<td>( I = 86.234(t_d + 0.056)^{-0.213} )</td>
<td></td>
</tr>
<tr>
<td>Model 8</td>
<td>( I = \frac{C}{t_d + m} )</td>
<td>86.234</td>
<td>0.078</td>
<td>0.193</td>
<td>1.102</td>
<td>( I = \frac{86234}{t_d + 0.078} )</td>
</tr>
<tr>
<td>Model 9</td>
<td>( I = \frac{C T_r^m}{t_d + a^{0.3333}} )</td>
<td>59.142</td>
<td>0.447</td>
<td>0.068</td>
<td>0.548</td>
<td>( I = \frac{59142 \cdot t_r^{0.548}}{t_d + 0.068} )</td>
</tr>
<tr>
<td>Model 10</td>
<td>( I = \frac{C}{t_d} )</td>
<td>1083.397</td>
<td>10.331</td>
<td>( I = \frac{1083397}{t_d} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 11</td>
<td>( I = \frac{C}{t_d + m^{1.3333}} )</td>
<td>176267707.329</td>
<td>58067360.415</td>
<td>( I = \frac{176267707.329}{t_d + 58067360.415} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 12</td>
<td>( I = \frac{C}{t_d + m} )</td>
<td>281.111</td>
<td>0.430</td>
<td>0.457</td>
<td>39.831</td>
<td>( I = \frac{281111 \cdot 0.457}{t_d + 39.831} )</td>
</tr>
<tr>
<td>Model 13</td>
<td>( I = \frac{C}{t_d} )</td>
<td>86.234</td>
<td>0.213</td>
<td>( I = \frac{86234}{t_d} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 14</td>
<td>( I = \frac{C T_r^m}{t_d} )</td>
<td>48.544</td>
<td>0.461</td>
<td>0.088</td>
<td>( I = \frac{48544}{t_d} )</td>
<td></td>
</tr>
<tr>
<td>Model 15</td>
<td>( I = \frac{C T_r^m}{t_d} )</td>
<td>21.429</td>
<td>0.691</td>
<td>0.213</td>
<td>( I = \frac{21429 \cdot t_r^{0.213}}{t_d} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Statistical evaluation of each of the models

<table>
<thead>
<tr>
<th>Evaluation</th>
<th>Total Err</th>
<th>AIC</th>
<th>SC</th>
<th>MSC</th>
<th>CD</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>214799.87</td>
<td>2584.27</td>
<td>2594.31</td>
<td>1.475</td>
<td>0.778</td>
<td>0.882</td>
</tr>
<tr>
<td>Model 2</td>
<td>820746.82</td>
<td>2865.77</td>
<td>2875.82</td>
<td>0.134</td>
<td>0.150</td>
<td>0.388</td>
</tr>
<tr>
<td>Model 3</td>
<td>826435.43</td>
<td>2867.22</td>
<td>2877.27</td>
<td>0.127</td>
<td>0.144</td>
<td>0.380</td>
</tr>
<tr>
<td>Model 4</td>
<td>192759.07</td>
<td>2561.53</td>
<td>2571.57</td>
<td>1.579</td>
<td>0.800</td>
<td>0.894</td>
</tr>
<tr>
<td>Model 5</td>
<td>197353.03</td>
<td>2566.48</td>
<td>2576.52</td>
<td>1.555</td>
<td>0.795</td>
<td>0.892</td>
</tr>
<tr>
<td>Model 6</td>
<td>818653.71</td>
<td>2865.24</td>
<td>2875.28</td>
<td>0.137</td>
<td>0.152</td>
<td>0.390</td>
</tr>
<tr>
<td>Model 7</td>
<td>818653.71</td>
<td>2865.24</td>
<td>2875.28</td>
<td>0.137</td>
<td>0.152</td>
<td>0.390</td>
</tr>
<tr>
<td>Model 8</td>
<td>827874.25</td>
<td>2867.59</td>
<td>2877.63</td>
<td>0.126</td>
<td>0.143</td>
<td>0.378</td>
</tr>
<tr>
<td>Model 9</td>
<td>238573.37</td>
<td>2606.31</td>
<td>2616.35</td>
<td>1.370</td>
<td>0.753</td>
<td>0.868</td>
</tr>
<tr>
<td>Model 10</td>
<td>849334.83</td>
<td>2872.96</td>
<td>2883.01</td>
<td>0.100</td>
<td>0.121</td>
<td>0.347</td>
</tr>
<tr>
<td>Model 11</td>
<td>1276827.82</td>
<td>2958.58</td>
<td>2968.62</td>
<td>-0.308</td>
<td>-0.322</td>
<td>0.567</td>
</tr>
<tr>
<td>Model 12</td>
<td>191899.83</td>
<td>2560.59</td>
<td>2570.63</td>
<td>1.593</td>
<td>0.802</td>
<td>0.896</td>
</tr>
<tr>
<td>Model 13</td>
<td>818653.71</td>
<td>2865.24</td>
<td>2875.28</td>
<td>0.142</td>
<td>0.137</td>
<td>0.396</td>
</tr>
<tr>
<td>Model 14</td>
<td>292732.98</td>
<td>2636.69</td>
<td>2646.71</td>
<td>1.161</td>
<td>0.696</td>
<td>0.834</td>
</tr>
<tr>
<td>Model 15</td>
<td>304671.97</td>
<td>2657.67</td>
<td>2667.71</td>
<td>1.125</td>
<td>0.685</td>
<td>0.827</td>
</tr>
</tbody>
</table>
4 CONCLUSION
It was concluded based on the findings that functions with duration of rainfall, return period, and empirically derived constants are the best expressions, which describe rainfall intensity in Makurdi best based on the values of MSC, CD, AIC and R. The three models 4, 5 and 12, are most the accurate models for the prediction of rainfall intensities in Makurdi. However, Model 12 is the most accurate for Makurdi rainfall intensities as against model 15 recommended by Isikwue et al., (2012).

REFERENCES