A Multivariate Regression Model for Predicting the 28days Flexural Strength of Lime-Cement Concrete Beam

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Abstract- In this study, a multivariate regression model for predicting the 28days flexural strength of lime-cement concrete prototype beam was developed. The response function is a multivariate function of the proportions of the component materials of concrete. A total of twenty mix ratios, consisting of water, Portland cement, hydrated lime, river sand and granite chipping were used in the prediction process. The first ten mix ratios were used for model development while the remaining ten mix ratios were used as check points for model validation. The model developed was tested for adequacy at 95% level of confidence using the t-statistic. Calculated t-value was -1.3342 and this was less than the critical t-value of 2.2622. Thus, the model was found to be adequate. An average percentage difference of 14.303% was observed between the model prediction and the experimental values. A visual basic program using the Visual studio 2015 software was developed based on the regression model. It was invoked to quicken the process of selecting the mix ratios of the component materials corresponding to any desired flexural strength value that falls within the region of experimentation and vice versa

Keywords- Concrete, flexural strength, multivariate regression model, response function

1 INTRODUCTION

A building is said to have collapsed when it no longer satisfies its main function of being safe, comfortable, stable and satisfying to its users (Bamgboye et al., 2019). This happens when it can no longer carry the load it was originally designed to support. The rate of structural failure and collapse of buildings in Nigeria in recent times is worrisome. These unfortunate developments have led to loss of many lives and damages of properties worth millions of dollars. Therefore, there is great need to construct durable and safe buildings (Fapohunda et al., 2020). A review previously conducted reported that materials such as cement, granite, steel, sand concrete and sandcrete blocks play a major role in the stability or collapse of many buildings in Nigeria Bamgboye et al. (2019). They stated that about 10% to 25% of all building collapse in Nigeria is as a result of poor-quality building materials of which low strength concrete due to improper mix proportioning is a major factor.

The structural failure and the eventual collapse of most buildings may be attributed to improper concrete mixture proportioning (Chendo & Obi, 2015; Oloyede, Omogun & Akiniare., 2010). In Nigeria, many on-site construction personnel use mix designs that are not supported by any experimental investigation or current design codes. They sometimes do this to either save cost or make more profit or to increase concrete workability for ease of workmanship. All these adversely affect the structural integrity of the building. Therefore, there is need to proffer and ensure that optimal mix designs for concrete production are achieved and adhered to for various construction needs.

The increase in demand for cement as a construction material in recent times is worrisome. This is because the more Portland cement is produced, the more the emission of CO₂ into the atmosphere. This CO₂ found in the earth's atmosphere is causing hotter and hotter temperatures of the globe. Some of the consequences are the loss of sea ice, rise in sea levels, heat waves leading to draught, change in temperature patterns and coastal erosion (NASA, 2021). In addition, a lot of energy is required in the manufacture of Portland cement making the cost of its production very high. This development has denied most Nigerians the opportunity to afford and own their own homes/shelter.

In this study, hydrated lime is introduced into the concrete mix as a partial replacement for Portland cement in order to reduce the cost of concrete production as well as reduce the high energy requirements for the production of Portland cement. In order to achieve excellent mix design of the concrete, a multivariate regression model has been developed to predict the flexural strength of the lime-cement concrete without the need for the classical laboratory experimental approach that is time consuming and labour demanding.

A mathematical model is a description of a system using mathematical concept and languages (Wikipedia, 2021). They are used to describe a system of variables and a set of equations that establishes relationship between the variables. The use of mathematical models in concrete mix design has proven to be successful. They are mathematically rigorous and can be used to define confidence intervals for predictions made. They provide insight into the key factors influencing strength properties of concrete through correlation analysis. Mathematical models evaluated on the basis of test data (experimental readings) are most reliable, accurate and scientific (Wilson, et al., 2019).

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Orie and Osadebe (2015) used the Scheffes mathematical optimization model to predict the flexural strength of mound soil cement blended concrete. They worked on concrete beams of size 150mm x 150mm x 750mm and developed a program that could predict the mix proportions for the model. Optimum mix ratio generated from their program was 1:1.59:0.46:3.34:0.53 (cement: fine aggregate: coarse aggregate: mound soil: water-cement ratio). This gave a flexural strength of 0.31N/mm². This optimal flexural strength value was compared to the flexural strength of conventional concrete made from mix ratio 1:2:4 (cement: fine aggregate: coarse aggregate) at 28 days. Its value was 1.93N/mm² at 0.5 water-cement ratio. The developed model was seen to be adequate but the mound soil cement blended concrete had flexural strength results that are lower than plain concrete but could still be used for structural purposes.

Dan-asabe et al. (2019), developed mathematical models for predicting the flexural strength, density and water absorption of Doum-palm-kankara clay filler hybrid composite. They used the Design Expert 10.0 software in carrying out their study and observed that the flexural strength data fitted best with the quartic model having an R-square value of 97.6% and an adjusted R² value of 93.6%. A number of other scholars have developed mathematical models for different types of unconventional concrete mixture optimization (Anyagwu & Ezeh, 2013; Ibearugbulem et al. (2013); Ndububa & Osadebe, 2007; Onwuka & Sule, 2017; Obam & Osadebe, 2006). However, their approach for model development is based on predetermined number of mix proportions. The approach used in this study is capable of modeling a set of mixes without predetermining the number of mix proportions.

In this work, mathematical model is developed to predict the flexural strength of hydrated lime-cement concrete beam using a least squares approach. A computer program written in Visual Basic language based on the model developed was invoked to quicken the prediction of the component mix ratios corresponding to the desired flexural strength value and vice versa. The use of hydrated lime as partial replacement for Portland cement as well as the proposed mathematical model for predicting mix ratios will go a long way in reducing the cost of producing cement for construction works and provide an easier and faster means of determining desired flexural strength values and their corresponding mix ratios.

2 MATERIALS AND METHODS

2.1 MATERIALS

(a) Cement: The cement used as binder for this study is ordinary Portland cement with properties conforming to BS EN 197, Part 1, 2000.

(b) Water: Water for the study was fresh, free of any form of organic matters and potable in nature.

(c) Aggregates: Fine aggregate: The fine aggregate was sharp river sand of nominal size 5mm. It was obtained from Otamiri River in Imo State and had a bulk density of

1656.022kg/m³. It was washed and sundried for three weeks before it was used for lime-cement concrete prototype beam production. Its bulk density was obtained according to the provisions of BS 882, part 2, 1992.

(d) Coarse aggregate: The granite chippings of nominal size 20mm were as coarse aggregate and obtained in bags from Obinze in Owerri, Imo State. It was properly washed and sundried to remove dirt and had a bulk density of 1706.225kg/m³.

(e) Hydrated lime: This was gotten from Okpella in Edo State and conformed to the specification of NIS 444-1 (2003).

2.1.1 Sample Preparation

Concrete components include water, Portland cement, hydrated lime; sand and gravel from crushed granite rock were used. Twenty (20) mix ratios were experimented (Awodiji, 2018). The first ten (10) mix ratios were used to formulate the model. While, the remaining ten (10) mix ratios were used to validate the model. Three 150mm x 150mm x 600mm prototype beams were produced for each mix ratio to get a total of sixty prototype beams. The beams were cured for twenty-eight days in water at room temperature before being subjected to testing. The mix ratios, corresponding component proportions and their twenty-eight (28) days flexural strength test results are presented in Table 1.

2.2 METHODS

The methods used in this study falls under the following classifications:

- i. Experimental
- ii. Model development
- iii. Statistical method
- iv. Program development

i. Experimental method: The beam specimen was subjected to flexure using a flexural testing machine in accordance to BS 1881, Part 118 (1983). The third point loading method was adopted and failure loads were determined. These failure loads were then used to obtain the flexural strength for the specimens by applying the formula in equation 1;

$$\text{Flexural strength (N/mm}^2\text{)} = NL/bd^2 \quad (1)$$

Where N is the failure load in Newton; b is the width of the specimen in "mm"; d is the depth of the specimen in "mm" and L is the length of the specimen in "mm".

ii. Development of the multivariate regression model: The five mixture components are water, cement, hydrated lime, river sand and granite chippings respectively. Let S be the total sum of the component ratios. Then the relative proportion of the ith component in the mixture is given by:

$$x_i = s_i / S \quad (2)$$

$$S = s_1 + s_2 + s_3 + s_4 + s_5 \quad (3)$$

Where x_i $i = (1, 2, 3, 4, 5)$ are proportions of water, cement, lime, sand and gravel respectively, s_1 = water, s_2 = cement, s_3 = lime, s_4 = sand, s_5 = gravel

From equation (2),

$$s_i = x_i * S \quad (i = 1, 2, 3, 4, 5) \quad (4)$$

Substituting for s_i in equation (3) gives:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1 \quad (5)$$

Equation (5) shows that the sum of the component mix ratios is unity.

The quadratic response function for a 5-component concrete mixture can be approximated by;

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_1^2 + \beta_7 X_2^2 + \beta_8 X_3^2 + \beta_9 X_4^2 + \beta_{10} X_5^2 + \beta_{11} X_1 X_2 + \beta_{12} X_1 X_3 + \beta_{13} X_1 X_4 + \beta_{14} X_1 X_5 + \beta_{15} X_2 X_3 + \beta_{16} X_2 X_4 + \beta_{17} X_2 X_5 + \beta_{18} X_3 X_4 + \beta_{19} X_3 X_5 + \beta_{20} X_4 X_5 + e \quad (6)$$

Equation (6) can be written in matrix form as:

$$Y = X\beta + E \quad (7)$$

Where E is the residual vector associated with the responses at different experimental points, β is the vector of the regression coefficients, X is an $n \times n$ matrix of mixture component proportions.

$$X = \begin{bmatrix} X_1 X_2 & X_3 X_4 X_5 & X_1^2 & X_2^2 X_3^2 X_4^2 X_5^2 X_1 X_2 & X_1 X_3 & X_1 X_4 X_1 X_5 & X_2 X_3 & X_2 X_4 \\ X_2 X_5 X_3 X_4 X_3 X_5 & X_4 X_5 \end{bmatrix} \quad (8)$$

Applying equation (7), the vector of the residuals associated with the responses is given by:

$$E = Y - X\beta \quad (9)$$

Equation (10) is obtained by taking a norm of equation (9) and then finding the vector of the unknown coefficient, β which minimizes equation (9). The norm of equation (9) is given by:

$$\|E\| = \|Y - X\beta\| \quad (10)$$

Equation (10) represents the least squares solution of equation (9).

In matrix form, equation (10) is given by:

$$E^T E = (Y - X\beta)^T (Y - X\beta) \quad (11)$$

Equation (11) yields:

$$E^T E = Y Y^T - 2\beta^T X^T Y^T + \beta^T X^T X \beta \quad (12)$$

The norm of the residual vector E is made small by differentiating equation (12) with respect to β and is given as:

$$d(E^T E) / d\beta = -2X^T Y + 2X^T X \beta = 0 \quad (13)$$

It now follows from equation (13) that:

$$\beta = (X^T X)^{-1} X^T Y \quad (14)$$

Equation (14) gives the coefficients β that minimizes the residual vector.

Where $(X^T X)^{-1}$ is an inverse of the moment matrix $(X^T X)$.

The moment matrix $(X^T X)$ is obtained by multiplying the transpose of the matrix of the component mix proportions by the matrix of the component mix proportions.

iii. **Statistical method:** Two statistical methods were adopted in determining the adequacy of the model predictions with respect to experimental results. They are the t-statistics and the percentage error methods. Their equations are respectively given in equations 15 and 16.

$$T = [DA * (N^{1/2})] / S \quad (15)$$

Where, $DA = (\sum D_i) / N$; $S = \sum S^2$; $S^2 = \sum (DA - D_i)^2 / (N - 1)$;

$D_i = YM - YE$

SE are the experimental responses; SM are the responses from the model and N represent the number of responses.

$$\% \text{ error} = \{SE - SM\} / SE * 100\% \quad (16)$$

iv. **Program development:** A computer program written in Visual Basic language was developed in order to quicken the prediction of desired flexural strength value and the corresponding mix ratios and vice versa. The program is interactive and can be used to predict the flexural strength of hydrated lime cement concrete beams and their corresponding mix ratios. The development of the program was carried out using the Visual studio 2015 software.

3 RESULTS AND DISCUSSION

The mix ratios and their corresponding flexural strength values are presented in Table 1. Mix ratios N1-N10 represent the trial mix ratios used for model development while, mix ratio C1-C10 represent the control mix ratios used for the model validation. The matrix of the component proportions using mix ratios N1-N10 was derived using equation (2), (3) and (8) respectively.

Table 1. Mix ratios and corresponding 28 days flexural strength values

Expt No.	Water	Cement	Lime	Sand	Gravel	Flexural strength (N/mm ²)
N1	0.6	0.9	0.1	3	6	2.28
N2	0.57	0.85	0.15	2	4	3.86
N3	0.55	0.8	0.2	2.5	5	3.62
N4	0.53	0.7	0.3	1.5	3	3.49
N5	0.5	0.6	0.4	1	2	2.96
N6	0.585	0.875	0.125	2.5	5	4.37
N7	0.575	0.85	0.15	2.75	5.5	3.91
N8	0.565	0.8	0.2	2.25	4.5	3.98
N9	0.55	0.75	0.25	2	4	3.23
N10	0.56	0.825	0.175	2.25	4.5	4.39
C1	0.55	0.775	0.225	1.75	3.5	4.26
C2	0.535	0.725	0.275	1.5	3	3.58
C3	0.54	0.75	0.25	2	4	2.77
C4	0.525	0.7	0.3	1.75	3.5	2.94
C5	0.515	0.65	0.35	1.25	2.5	2.67
C6	0.586	0.875	0.125	2.5	5	4.51
C7	0.575	0.85	0.15	2.75	5.55	3.27
C8	0.55	0.775	0.225	1.75	3.55	4.02
C9	0.525	0.7	0.3	1.75	3.55	2.82
C10	0.517	0.65	0.35	1.25	2.5	2.7

Source: (Awodiji, 2018)

The vector of responses of the first ten mix ratios based on Table 1 is given as:

$$Y = [2.28 \ 3.86 \ 3.62 \ 3.49 \ 2.96 \ 4.37 \ 3.91 \ 3.98 \ 3.23 \ 4.39] \quad (17)$$

The matrices of the component proportions (X) and its transpose (X^T) are given in equation (18) and (19) respectively. The results of moment matrix ($X^T X$) and moment matrix inverse ($(X^T X)^{-1}$) are presented in equation (20) and (21) respectively. The multiplication of matrices (X^T and X) was achieved using 'Mmult' in Microsoft Excel.

X =

0.056	0.084	0.009	0.283	0.566	0.003	0.007	0.000	0.080	0.320	0.004	0.000	0.016	0.032	0.000	0.024	0.048	0.002	0.005	0.160
0.075	0.112	0.019	0.264	0.528	0.005	0.012	0.000	0.069	0.279	0.008	0.001	0.019	0.039	0.002	0.029	0.059	0.005	0.010	0.139
0.060	0.088	0.022	0.276	0.552	0.003	0.007	0.000	0.076	0.305	0.005	0.001	0.016	0.033	0.002	0.024	0.048	0.006	0.012	0.152
0.087	0.116	0.049	0.248	0.497	0.007	0.013	0.002	0.061	0.247	0.010	0.004	0.021	0.043	0.005	0.028	0.057	0.012	0.024	0.123
0.111	0.133	0.088	0.222	0.444	0.012	0.017	0.007	0.049	0.197	0.014	0.009	0.024	0.049	0.011	0.029	0.059	0.019	0.039	0.098
0.064	0.096	0.013	0.275	0.550	0.004	0.009	0.000	0.075	0.302	0.006	0.000	0.017	0.035	0.001	0.026	0.053	0.003	0.007	0.151
0.058	0.086	0.015	0.279	0.559	0.003	0.007	0.000	0.078	0.313	0.005	0.000	0.016	0.032	0.001	0.024	0.048	0.004	0.008	0.156
0.067	0.096	0.024	0.270	0.541	0.004	0.009	0.000	0.073	0.292	0.006	0.001	0.018	0.036	0.002	0.026	0.052	0.006	0.013	0.146
0.072	0.099	0.033	0.264	0.529	0.005	0.009	0.001	0.070	0.280	0.007	0.002	0.019	0.038	0.003	0.026	0.052	0.008	0.017	0.140
0.067	0.099	0.021	0.270	0.541	0.004	0.009	0.000	0.073	0.293	0.006	0.001	0.018	0.036	0.002	0.026	0.053	0.005	0.011	0.146

(18)

 $X^T =$

0.0566	0.0753	0.0608	0.0879	0.1111	0.0644	0.0585	0.0679	0.0728	0.0674
0.0849	0.1123	0.0884	0.1161	0.1333	0.0963	0.0865	0.0962	0.0993	0.0993
0.0094	0.0198	0.0221	0.0498	0.0889	0.0138	0.0153	0.0241	0.0331	0.0211
0.2830	0.2642	0.2762	0.2488	0.2222	0.2752	0.2799	0.2706	0.2649	0.2708
0.5660	0.5284	0.5525	0.4975	0.4444	0.5504	0.5598	0.5412	0.5298	0.5415
0.0032	0.0057	0.0037	0.0077	0.0123	0.0041	0.0034	0.0046	0.0053	0.0045
0.0072	0.0126	0.0078	0.0135	0.0178	0.0093	0.0075	0.0093	0.0099	0.0099
0.0001	0.0004	0.0005	0.0025	0.0079	0.0002	0.0002	0.0006	0.0011	0.0004
0.0801	0.0698	0.0763	0.0619	0.0494	0.0757	0.0783	0.0732	0.0702	0.0733
0.3204	0.2792	0.3053	0.2475	0.1975	0.3029	0.3134	0.2929	0.2807	0.2932
0.0048	0.0085	0.0054	0.0102	0.0148	0.0062	0.0051	0.0065	0.0072	0.0067
0.0005	0.0015	0.0013	0.0044	0.0099	0.0009	0.0009	0.0016	0.0024	0.0014
0.0160	0.0199	0.0168	0.0219	0.0247	0.0177	0.0164	0.0184	0.0193	0.0183
0.0320	0.0398	0.0336	0.0437	0.0494	0.0354	0.0327	0.0367	0.0386	0.0365
0.0008	0.0022	0.0020	0.0058	0.0119	0.0013	0.0013	0.0023	0.0033	0.0021
0.0240	0.0297	0.0244	0.0289	0.0296	0.0265	0.0242	0.0260	0.0263	0.0269
0.0481	0.0593	0.0488	0.0578	0.0592	0.0530	0.0484	0.0521	0.0526	0.0538
0.0027	0.0052	0.0061	0.0124	0.0198	0.0038	0.0043	0.0065	0.0088	0.0057
0.0053	0.0105	0.0122	0.0248	0.0395	0.0076	0.0086	0.0130	0.0175	0.0114
0.1602	0.1396	0.1526	0.1238	0.0987	0.1515	0.1567	0.1464	0.1403	0.1466

(19)

$$(X^T X) =$$

0.0547	0.0754	0.0249	0.1893	0.3785	0.0043	0.0081	0.0013	0.0498	0.1993	0.0059	0.0022	0.0141	0.0281	0.0029	0.0195	0.0391	0.0062	0.0123	0.0997
0.0754	0.1046	0.0330	0.2666	0.5331	0.0059	0.0111	0.0017	0.0705	0.2820	0.0080	0.0029	0.0195	0.0390	0.0037	0.0273	0.0545	0.0082	0.0164	0.1410
0.0249	0.0330	0.0139	0.0752	0.1504	0.0022	0.0037	0.0009	0.0192	0.0767	0.0029	0.0013	0.0062	0.0123	0.0017	0.0082	0.0164	0.0033	0.0066	0.0383
0.1893	0.2666	0.0752	0.7083	1.4165	0.0140	0.0273	0.0033	0.1896	0.7583	0.0195	0.0061	0.0499	0.0996	0.0082	0.0705	0.1410	0.0192	0.0383	0.3791
0.3785	0.5331	0.1504	1.4165	2.8329	0.0281	0.0546	0.0066	0.3791	1.5166	0.0391	0.0123	0.0997	0.1993	0.0165	0.1410	0.2820	0.0384	0.0767	0.7583
0.0043	0.0059	0.0022	0.0140	0.0281	0.0004	0.0007	0.0001	0.0036	0.0145	0.0005	0.0002	0.0011	0.0022	0.0003	0.0015	0.0030	0.0005	0.0011	0.0073
0.0081	0.0111	0.0037	0.0273	0.0546	0.0007	0.0012	0.0002	0.0072	0.0286	0.0009	0.0003	0.0021	0.0041	0.0004	0.0029	0.0057	0.0009	0.0018	0.0143
0.0013	0.0017	0.0009	0.0033	0.0066	0.0001	0.0002	0.0001	0.0008	0.0032	0.0002	0.0001	0.0003	0.0006	0.0001	0.0004	0.0008	0.0002	0.0004	0.0016
0.0498	0.0705	0.0192	0.1896	0.3791	0.0036	0.0072	0.0008	0.0509	0.2036	0.0051	0.0015	0.0132	0.0264	0.0021	0.0187	0.0375	0.0049	0.0098	0.1018
0.1993	0.2820	0.0767	0.7583	1.5166	0.0145	0.0286	0.0032	0.2036	0.8146	0.0204	0.0061	0.0528	0.1055	0.0083	0.0749	0.1499	0.0197	0.0394	0.4073
0.0059	0.0080	0.0029	0.0195	0.0391	0.0005	0.0009	0.0002	0.0051	0.0204	0.0007	0.0003	0.0015	0.0030	0.0003	0.0021	0.0041	0.0007	0.0014	0.0102
0.0022	0.0029	0.0013	0.0061	0.0123	0.0002	0.0003	0.0001	0.0015	0.0061	0.0003	0.0001	0.0005	0.0011	0.0002	0.0007	0.0014	0.0003	0.0006	0.0031
0.0141	0.0195	0.0062	0.0499	0.0997	0.0011	0.0021	0.0003	0.0132	0.0528	0.0015	0.0005	0.0037	0.0073	0.0007	0.0051	0.0102	0.0015	0.0031	0.0264
0.0281	0.0390	0.0123	0.0996	0.1993	0.0022	0.0041	0.0006	0.0264	0.1055	0.0030	0.0011	0.0073	0.0146	0.0014	0.0102	0.0204	0.0031	0.0062	0.0527
0.0029	0.0037	0.0017	0.0082	0.0165	0.0003	0.0004	0.0001	0.0021	0.0083	0.0003	0.0002	0.0007	0.0014	0.0002	0.0009	0.0018	0.0004	0.0008	0.0041
0.0195	0.0273	0.0082	0.0705	0.1410	0.0015	0.0029	0.0004	0.0187	0.0749	0.0021	0.0007	0.0051	0.0102	0.0009	0.0071	0.0143	0.0021	0.0041	0.0375
0.0391	0.0545	0.0164	0.1410	0.2820	0.0030	0.0057	0.0008	0.0375	0.1499	0.0041	0.0014	0.0102	0.0204	0.0018	0.0143	0.0286	0.0041	0.0083	0.0749
0.0062	0.0082	0.0033	0.0192	0.0384	0.0005	0.0009	0.0002	0.0049	0.0197	0.0007	0.0003	0.0015	0.0031	0.0004	0.0021	0.0041	0.0008	0.0016	0.0099
0.0123	0.0164	0.0066	0.0383	0.0767	0.0011	0.0018	0.0004	0.0098	0.0394	0.0014	0.0006	0.0031	0.0062	0.0008	0.0041	0.0083	0.0016	0.0032	0.0197
0.0997	0.1410	0.0383	0.3791	0.7583	0.0073	0.0143	0.0016	0.1018	0.4073	0.0102	0.0031	0.0264	0.0527	0.0041	0.0375	0.0749	0.0099	0.0197	0.2036

(20)

$$(X^T X)^{-1} =$$

4548.723	-5286.340	-6492.494	-232.775	2177.143	-423.748	3358.573	15292.073	5985.023	-6720.372	-2378.387	1636.589	4563.652	-4729.066	9748.235	-3345.555	3455.195	3780.649	3454.239	4447.563
-5286.340	2239.248	1289.995	5859.977	60.791	-3570.448	10661.061	-5764.789	10664.525	-1069.761	1408.711	12073.236	12824.231	4110.643	663.331	6613.479	-5289.446	-6478.998	-6368.522	-3283.570
-6492.494	1289.995	-6867.394	7749.406	1301.222	-6149.263	-8138.041	11341.112	-3123.768	-6453.917	919.275	14043.044	25776.015	7888.903	12783.862	6196.181	13376.250	-1922.751	-481.434	-2503.911
-232.775	5859.977	7749.406	13088.177	-916.170	-316.497	9785.857	-4142.054	3511.925	9461.955	12723.429	16235.355	12425.683	-476.896	-8778.798	781.927	2073.356	6267.960	-1546.298	2786.370
2177.143	60.791	1301.222	-916.170	-2729.162	6479.953	4942.851	336.323	4587.538	3015.920	-1002.878	-5590.417	11997.038	-1753.037	-1606.612	-2582.737	3927.552	2930.406	2194.700	2461.393
-423.748	-3570.448	-6149.263	-316.497	6479.953	691.158	6018.584	9637.760	4274.146	-9825.031	1573.768	-2700.379	-665.122	-5860.455	2710.530	-4949.062	-263.588	-3525.541	10391.859	-1028.198
3358.573	-10661.061	-8138.041	9785.857	4942.851	6018.584	-5478.747	10173.775	11273.036	18768.408	12162.930	889.895	2089.599	-8029.357	1399.993	-5627.799	6308.447	-1355.205	6389.662	1957.147
15292.073	-5764.789	11341.112	-4142.054	336.323	9637.760	10173.775	33091.641	3887.423	8384.192	26811.260	27076.162	64021.546	29183.556	36055.130	26335.251	29656.033	-9380.860	11132.105	-9678.845
5985.023	-10664.525	-3123.768	3511.925	4587.538	4274.146	11273.036	3887.423	4705.180	-3733.951	11650.324	13054.856	19997.014	-6122.013	961.077	-8329.372	8717.290	2308.877	3180.141	-12760.692
-6720.372	-1069.761	-6453.917	9461.955	3015.920	-9825.031	18768.408	8384.192	-3733.951	14702.552	1675.405	25409.356	35869.293	7037.227	9607.902	5586.595	-9448.100	-8993.419	-2333.839	5252.063
-2378.387	1408.711	919.275	12723.429	-1002.878	1573.768	12162.930	26811.260	11650.324	1675.405	21510.707	-3161.031	7986.122	8375.115	8172.915	8794.922	-2837.965	11108.338	-2541.294	14031.698
1636.589	12073.236	14043.044	16235.355	-5590.417	-2700.379	889.895	27076.162	13054.856	25409.356	-3161.031	10106.176	12016.864	4497.128	-2788.162	7124.294	-1555.256	-1200.540	15721.435	-3207.610
4563.652	12824.231	25776.015	12425.683	11997.038	-665.122	2089.599	64021.546	19997.014	35869.293	7986.122	12016.864	27512.690	-1481.446	26584.576	2899.832	5451.516	-9812.729	16015.598	-5767.058
-4729.066	4110.643	7888.903	-476.896	-1753.037	-5860.455	-8029.357	29183.556	-6122.013	7037.227	8375.115	4497.128	-1481.446	3897.813	-8047.869	6301.282	429.242	-2772.636	-7058.606	-6254.942
9748.235	663.331	12783.862	-8778.798	-1606.612	2710.530	1399.993	36055.130	961.077	9607.902	8172.915	-2788.162	26584.576	-8047.869	19043.462	12602.205	10734.035	-9321.263	-6700.665	99.368
-3345.555	6613.479	6196.181	781.927	-2582.737	-4949.062	-5627.799	26335.251	-8329.372	5586.595	8794.922	7124.294	2899.832	6301.282	12602.205	-1486.223	-5767.655	-3618.221	-7447.360	-1184.297
3455.195	-5289.446	13376.250	2073.356	3927.552	-263.588	6308.447	29656.033	8717.290	-9448.100	-2837.965	1555.256	5451.516	429.242	10734.035	-5767.655	-1971.360	1548.448	12300.875	-471.880
3780.649	-6478.998	-1922.751	6267.960	2930.406	-3525.541	-1355.205	-9380.860	2308.877	-8993.419	11108.338	-1200.540	-9812.729	-2772.636	-9321.263	-3618.221	1548.448	-2787.206	5858.424	-1150.617
3454.239	-6368.522	-481.434	-1546.298	2194.700	10391.859	6389.662	11132.105	3180.141	-2333.839	-2541.294	15721.435	16015.598	-7058.606	-6700.665	-7447.360	12300.875	5858.424	6506.945	21.856
4447.563	-3283.570	-2503.911	2786.370	2461.393	-1028.198	1957.147	-9678.845	12760.692	5252.063	14031.698	-3207.610	-5767.058	-6254.942	99.368	-1184.297	-471.880	-1150.617	21.856	-15778.165

(21)

The $X^T Y$ vector is obtained by multiplying equation (19) by equation (17) as:

$$X^T Y = \begin{bmatrix} 2.59 & 3.65 & 1.03 & 9.61 & 19.22 & 0.19 \\ 0.38 & 0.05 & 2.57 & 10.27 & 0.27 & 0.08 & 0.68 \\ 1.36 & 0.11 & 0.96 & 1.93 & 0.26 & 0.53 & 5.14 \end{bmatrix} \quad (22)$$

Where Y vector is given in equation (7)

The coefficient vector β is obtained from equation (14) by multiplying equation (21) by equation (22) using 'Mmult' in Microsoft Excel as:

$$\beta = \begin{bmatrix} -13 & 39.2 & 13.9 & 4.6 & -25.5 & -2.6 & -15.1 & -48.1 & -22.9 & 29.5 & -43.8 & 49.1 & 29.9 & 47.8 & -29.1 & 37.6 & -6.2 & -12.8 & -36.4 & 27.3 \end{bmatrix} \quad (23)$$

Substituting equation (23) into equation (6) gives the quadratic equation for the flexural strength of the selected mix ratios. This is given as:

$$y = -13x_1 + 39.2x_2 + 13.3x_3 + 4.6x_4 - 25.5x_5 - 2.6x_1^2 - 15.1x_2^2 - 48.1x_3^2 - 22.9x_4^2 + 29.5x_5^2 - 43.8x_1x_2 + 49.1x_1x_3 + 29.9x_1x_4 + 47.8x_1x_5 - 29.1x_2x_3 + 37.6x_2x_4 - 6.2x_2x_5 - 12.8x_3x_4 - 36.4x_3x_5 + 27.3x_4x_5 \quad (24)$$

Equation (24) is the mathematical model that predicts the flexural strength of lime-cement concrete for the selected mix ratios based on quadratic response surface.

3.1 TEST OF ADEQUACY OF THE NETWORK

The observed and predicted values of flexural strength are compared in Table 2 using the percentage error.

Table 2. Comparison of observed and predicted values of flexural strength

Mix no.	Observed values	Predicted values	%Error
C1	4.26	3.849	9.65
C2	3.58	3.709	3.60
C3	2.77	3.469	25.23
C4	2.94	3.304	12.38
C5	2.67	3.323	24.46
C6	4.51	3.862	14.37
C7	3.27	3.558	8.81
C8	4.02	3.833	4.65
C9	2.82	3.294	16.81
C10	2.7	3.323	23.07
Av. % error =			14.303

The results obtained from equation (19) are tested for adequacy at 95% level of confidence against the control points using t-statistic as shown in Table 3:

Table 3. T-statistic test of flexural strength model based on quadratic function

	Experiment	Predicted
Mean	3.354	3.5524
Variance	0.4836	0.0586
Observations	10	10
Pearson Correlation	0.9534	
Hypothesized Mean Difference	0	
df	9	
t Stat	-1.3342	
P(T<=t) one-tail	0.1075	
t Critical one-tail	1.8331	
P(T<=t) two-tail	0.2149	
t Critical two-tail	2.2622	

Source: Microsoft Excel (2008)

From Table 3, it can be seen that the calculated value of t (-1.3342) is less than the critical t -value (2.2622). Hence, the null hypothesis is accepted and the model is adequate and can be used to predict the flexural strength of hydrated lime-cement concrete.

4 CONCLUSION

A multivariate regression model has been developed to predict the flexural strength of hydrated lime-cement concrete based on least squares approach. The reliability of the model was investigated at 5% level of significance using t -statistic and was found to be adequate. A computer program written in Visual Basic was invoked to quicken the prediction of desired flexural strength value and the corresponding mix ratios and vice versa. The Visual Basic program developed is found to be interactive and can be used to predict with reasonable accuracy, the flexural strength of hydrated lime-cement concrete beams and the corresponding mix ratios.

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